NASA CR-145062

(NASA-CR-145062) FRACTURE OF COMPOSITE PANELS (Lehigh Univ.) 22 p HC $3.50 CSCL 11D
N76-30297
Unclas
G3/24 50402

FRACTURE OF COMPOSITE PANELS

BY

F. ERDOGAN

AND

M. BAKIOGLU

FEBRUARY 1976

LEHIGH UNIVERSITY

BETHLEHEM, PENNSYLVANIA

THE NATIONAL AERONAUTICS AND

SPACE ADMINISTRATION

GRANT NO. NGR 39-007-011

FRACTURE OF COMPOSITE PANELS

BY

F. ERDOGAN

AND

M. BAKIOGLU

FEBRUARY 1976

LEHIGH UNIVERSITY
BETHLEHEM, PENNSYLVANIA

THE NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION
GRANT NO. NGR 39-007-011

FRACTURE OF COMPOSITE PANELS

F. Erdogan[1] and M. Bakioglu[2]

ABSTRACT: The fracture problem in panels which consist of periodically arranged load-carrying and buffer strips of different materials is considered. The main emphasis in the study is placed on the problem of a crack terminating at and crossing the interfaces and on the stress-free end problem. The problem is formulated in terms of a system of singular integral equations and numerical solutions are obtained for certain material combinations. With the study of possible crack propagation and delamination in mind, certain stress intensity factors are defined and calculated. A main result of the study is that when the crack touches or intersects a bimaterial interface, the stress state has no longer the standard square root singularity and, to study further propagation of the crack, the conventional fracture models need to be modified or new models need to be developed.

KEY WORDS: Composite materials, fracture, crack propagation, delamination, stress singularity, bonded layers, stress-free end.

In structural design with high strength composite sheet materials the use of relatively low stiffness and high toughness buffer strips oriented parallel to the main load-carrying laminates has been under investigation for some time [1,2]. The practical objective of this design procedure is to improve the fatigue crack propagation and arrest characteristics of the structure. The fracture process may start as a fatigue crack initiation at a local imperfection in the load-carrying laminate. Under repeated loading it may be possible for the propagating crack to reach the interface and to enter the adjacent buffer strips. Assuming that the fatigue and fracture characteristics of both materials are known, for studies relating to fatigue life and structural integrity, it would be necessary to have a reliable analysis of the problem. Particularly in fatigue crack growth studies, it would be very useful to have a technique for calculating the corresponding stress intensity factors.

After the crack propagating in the first strip touches the interface, further fracture propagation would take place along the plane for which the ratio of the local load factor (or crack driving force) representing the intensity of the applied loads and the geometry to the corresponding strength parameter of the two materials or the interface bond is maximum. In this case, since the stress state around the crack tip does not have the standard square root singularity, the conventional fatigue and fracture models cannot be used to study the problem. To

(1) Professor of Mechanics, Lehigh University, Bethlehem, PA 18015.
(2) Lecturer, Department of Civil Engineering, Istanbul Technical University, Istanbul, Turkey.

do this, in addition to having the correct analysis of the problem it may be necessary to make certain modifications in the existing fatigue and fracture models.

In the actual structures, particularly in those which consist of fiber reinforced laminates, generally the strips are anisotropic [1]. However, in this study, largely for reasons of analytical expediency, both materials are assumed to be isotropic and linearly elastic. The basic formulation of the problem of periodically arranged bonded dissimilar strips and its solution for cracks growing in one set of strips and terminating at the interfaces were given in [2] (Figure 1a). In this paper the emphasis is on the propagation of the crack in the second medium after crossing the interface and on the "stress-free end problem" of bonded layered materials (Figures 1a and 1b).

Formulation of the Problem

The general problem under consideration is described in Figure 1. It is a two-dimensional elastostatics problem for a composite medium which consists of periodically arranged two sets of bonded dissimilar strips having collinear cracks as shown in Figures 1a and 1b. The medium may be under symmetric uniform normal tractions in x, y, and (in plane strain case) z directions, or under internal stresses due to the difference between curing and operating temperatures. Under any combination of these loads the problem giving the homogeneous stress state in the composite medium in the absence of cracks may be solved and the stress components on $y=0$ plane may be obtained without any difficulty. Let these stresses be

$$\begin{aligned} \sigma^{\infty}_{1yy}(x,0) &= p_1(x) \quad , \quad \sigma^{\infty}_{1xy}(x,0) = 0 \quad , \\ \sigma^{\infty}_{2yy}(x,0) &= p_2(x) \quad \quad \sigma^{\infty}_{2xy}(x,0) = 0 \quad . \end{aligned} \tag{1}$$

The solution of the original crack problem is obtained by adding to the homogeneous solution the solution found from a perturbation problem in which the self-equilibrating crack surface tractions are the only external loads. From the view point of fracture the important problem is the perturbation problem which may be solved by considering the standard field equations for the strips 1 and 2 under the following boundary and continuity conditions:

$$u_1(0,y) = u_2(0,y) \ , \ v_1(0,y) = v_2(0,y) \ , \ (0\le y<\infty) \ ,$$

$$\sigma_{1xx}(0,y) = \sigma_{2xx}(0,y) \ , \ \sigma_{1xy}(0,y) = \sigma_{2xy}(0,y) \ , \ (0\le y<\infty) \ ,$$

$$u_1(-h_1,y) = 0 \ , \ \sigma_{1xy}(-h_1,y) = 0 \ , \ (0<y<\infty) \ ,$$
$$u_2(h_2,y) = 0 \ , \ \sigma_{2xy}(h_2,y) = 0 \ , \ (0<y<\infty) \ ,$$
$$\sigma_{1xy}(x,0) = 0 \ , \ (-h_1\le x\le 0) \ , \ \sigma_{2xy}(x,0) = 0 \ , \ (0\le x\le h_2) \ ,$$
$$\sigma_{1yy}(x,0) = -p_1(x) \ , \ (-h_1\le x<a-h_1) \ , \ v_1(x,0) = 0 \ , \ (a-h_1<x<0) \ ,$$
$$\sigma_{2yy}(x,0) = -p_2(x) \ , \ (h_2-b<x<h_2-c) \ , \ v_2(x,0) = 0 \ , \ (0<x<h_2-b, \ h_2-c<x<h_2) \ ,$$
$$\sigma_{kyy}(x,\infty) = 0 \ , \ \sigma_{kxy}(x,\infty) = 0 \ , \ (k=1,2) \ , \tag{2}$$

where, because of symmetry, only one quarter of a strip from each set is considered, u_k, v_k, σ_{kij} are the displacement and stress components in the usual notation ($k=1,2$; $i,j=x,y$), and the dimensions h_1, h_2, a, b, and c are shown in Figure 1. Defining

$$x_1 = x + h_1 \ , \ x_2 = x - h_2 \ , \ y_1 = y = y_2 \tag{3}$$
$$G_i(x_i) = \frac{\partial}{\partial x_i} v_i(x_i,0) \ , \ (i=1,2) \tag{4}$$

and using the technique described in [2], the perturbation problem may be reduced to the following system of singular integral equations:

$$\int_{-a}^{a}\frac{1}{t-x_1} G_1(t)dt + \int_{-a}^{a}k_{11}(x_1,t)G_1(t)dt + \int_{c}^{b}k_{12}(x_1,t)G_2(t)dt = -\frac{\pi(1+\kappa_1)}{4\mu_1} p_1(x_1-h_1) \ , \ -a<x<a \ ,$$

$$\int_{c}^{b}(\frac{1}{t-x_2} + \frac{1}{t+x_2})G_2(t)dt + \int_{a}^{a}k_{21}(x_2,t)G_1(t)dt + \int_{c}^{b}k_{22}(x_2,t)G_2(t)dt = -\frac{\pi(1+\kappa_2)}{4\mu_2} p_2(x_2+h_2) \ , \ b<x_2<b \ , \tag{5}$$

where μ_i, κ_i, $(i=1,2)$ are the elastic constants with $\kappa_i = (3-\nu_i)/(1+\nu_i)$ for generalized plane stress, $\kappa_i = 3-4\nu_i$ for plane strain, ν_i being one Poisson's ratio. The kernels $k_{ij}(x_i,t)$, $(i,j=1,2)$ are given by

$$k_{i1}(x_i,t) = \int_0^\infty K_{i1}(x_i,t,s)e^{-s(h_1-t)}ds \ ,$$
$$k_{i2}(x_i,t) = \int_0^\infty [K_{i2}(x_i,t,s)e^{-s(h_2-t)} - K_{i2}(x_i,-t,s)e^{-s(h_2+t)}]ds \ (i=1,2) \tag{6}$$

where the functions K_{ij} are defined in the Appendix B of [2] and will not be repeated in this paper. The index of the integral equations (5) is +1 [3]. Therefore the general solution of (5) will contain two arbitrary constants [3]

which may be determined by using the following single-valuedness conditions:

$$\int_{-a}^{a} G_1(t)dt = 0 \quad , \quad \int_{c}^{b} G_2(t)dt = 0 \quad . \tag{7}$$

For cracks imbedded in the strips, i.e., for $a<h_1$ and $0\le c\le b<h_2$, the solution of (5) is of the form

$$G_1(t) = g_1(t)(a^2-t^2)^{-\frac{1}{2}} \; , \; -a<t<a \; ,$$

$$G_2(t) = g_2(t)[(b-t)(t-c)]^{-\frac{1}{2}} \; , \; c<t<b \; , \tag{8}$$

where g_1 and g_2 are bounded in their respective closed domains. The unknown functions g_1 and g_2 may be determined from (5) and (7) in a straightforward way by using the technique described in [4]. For this case the stress intensity factors at the crack tips are defined and are expressed in terms of G_1 and G_2 as follows:

$$k_a = \lim_{x_1\to a} [2(x_1-a)]^{\frac{1}{2}}\sigma_{1yy}(x_1,0) = -\frac{4\mu_1}{1+\kappa_1} \lim_{x_1\to a} [2(a-x_1)]^{\frac{1}{2}}G_1(x_1) \; ,$$

$$k_b = \lim_{x_2\to b} [2(x_2-b)]^{\frac{1}{2}}\sigma_{2yy}(x_2,0) = -\frac{4\mu_2}{1+\kappa_2} \lim_{x_2\to b} [2(b-x_2)]^{\frac{1}{2}}G_2(x_2) \; ,$$

$$k_c = \lim_{x_2\to c} [2(c-x_2)]^{\frac{1}{2}}\sigma_{2yy}(x_2,0) = \frac{4\mu_2}{1+\kappa_2} \lim_{x_2\to c} [2(x_2-c)]^{\frac{1}{2}}G_2(x_2) \; . \tag{9}$$

The case of a crack touching the interface, i.e., $a=h_1$, was treated in [2] where it was shown that

$$G_1(t) = g_1(t)(h_1^2-t^2)^{-\beta} \; ,$$

$$k_a = \lim_{x_2\to -h_2} \sqrt{2}\,(x_2+h_2)^{\beta}\sigma_{2yy}(x_2,0) = -\,2\mu_o \lim_{x_1\to h_1} \lim\sqrt{2}\,(h_1-x_1)^{\beta}G_1(x_1) \; ,$$

$$\mu_o = \frac{\mu_1\mu_2}{\sin\pi\beta}\left(\frac{1+2\alpha_1(1-\beta)}{\mu_1+\kappa_1\mu_2} + \frac{1-2\alpha_1(1-\beta)}{\mu_2+\kappa_2\mu_1}\right) \; ,$$

$$2\cos\pi\beta + 4\alpha_2(\beta-1)^2 - (\alpha_1+\alpha_2) = 0 \; , \quad 0<\beta<1$$

$$\alpha_1 = (\kappa_1\mu_2-\kappa_2\mu_1)/(\mu_2+\kappa_2\mu_1) \quad , \quad \alpha_2 = (\mu_2-\mu_1)/(\mu_1+\kappa_1\mu_2) \; . \tag{10}$$

where β is the power of stress singularity which is real for all material combinations, and k_a is the "stress intensity factor".

Crack Crossing the Interface

In the integral equations (5), if $a<h_1$, $b<h_2$, and $0\le c<b$, it was shown

that the kernels k_{ij} are bounded in their respective closed domains [2]. Referring to Figure 1a and 1b, if we let $a = h_1$, $b = h_2$, $0<c<h_2$, it is seen that the problem becomes one of a crack crossing the interface for which equations (5) are still valid. However, in this case following the procedure outlined in [2] and [5], it can be shown that as x_i, $(i = 1,2)$ and t go to the interface together the kernels k_{ij} $(i,j = 1,2)$ become unbounded. The singular parts, $k_{ijs}(x_i,t)$, of these kernels may be separated by examining the asymptotic behavior of the integrals given by (6). The kernels k_{ijs} are given in the Appendix. Together with the kernels $(t-x_i)^{-1}$, k_{ijs} constitute a set of generalized Cauchy kernels. In this problem the singular behavior of the solution can be examined by expressing

$$G_1(t) = g_1(t)(h_1^2-t^2)^{-\gamma} \quad , \; G_2(t) = g_2(t)(h_2-t)^{-\gamma}(t-c)^{-\eta} \; , \; 0<\mathrm{Re}(\eta,\gamma)<1 \; , \tag{11}$$

where g_1 and g_2 are again bounded. The characteristic equations to determine the constants η and γ are obtained from the dominant part of (5) by using the complex function technique [3-6] as follows:

$$\begin{aligned} &\cot\pi\eta = 0 \; , \\ &[2\cos\pi\gamma + 4\alpha_2(1-\gamma)^2-\alpha_1-\alpha_2][2\cos\pi\gamma + 4\alpha_4(1-\gamma)^2 \\ &-\alpha_3-\alpha_4] - [2\gamma(\alpha_1-\alpha_2) + 2-\alpha_1 + 3\alpha_2][2\gamma(\alpha_3-\alpha_4) \\ &+ 2-\alpha_3 + 3\alpha_4] = 0 \; . \end{aligned} \tag{12}$$

where the constant $(\alpha_i, i = 1,..,4)$ are defined in the Appendix. From (12) it is found that $\eta = 0.5$, which is the well-known result, and for all material combinations there is only one root γ satisfying $0<\mathrm{Re}(\gamma)<1$ which is always real. The same analysis also gives the following relationship between the end values of the two bounded functions g_1 and g_2:

$$g_2(h_2) = - \frac{(h_2-c)^{\frac{1}{2}}}{(2h_1)^{\gamma}} \; \frac{2\cos\pi\gamma \; + 4\alpha_2(1-\gamma)^2-\alpha_1-\alpha_2}{2\gamma(\alpha_1-\alpha_2) + 2 - \alpha_1+3\alpha_2} \, g_1(h_1) \tag{13}$$

Equations (12) and (13) are identical to those obtained for the semi-infinite planes in [5].

A unique solution of (5) in this case too requires two additional conditions. One condition is given by the single-valuedness of the crack surface displacements which, referring to Figure 1b and the definitions (3) and (4), may be expressed as

$$\int_c^{h_2} G_2(t_2)dt_2 + \int_{-h_1}^{h_1} G_1(t_1)dt_1 + \int_{h_2}^{-c} G_2(t_2)dt_2 = 0 \tag{14}$$

The second condition is provided by (13).

In this problem the stress state is singular at the crack tip $x_2 = c$, $y_2 = 0$ as well as at the point of intersection of the crack and the interface, $x_1 = \mp h_1$, $y_1 = 0$. By examining the stresses around these points, from the solution of the problem it can be shown that the stress intensity factors are related to the bounded functions g_1 and g_2 as follows [5].

$$k_c = \lim_{x_2 \to c} [2(c-x_2)]^{\frac{1}{2}} \sigma_{2yy}(x_2,0) = -\frac{4\mu_2}{1+\kappa_2} \frac{\sqrt{2}}{(h_2-c)^{\gamma}} g_2(c) \;,$$

$$k_{xx} = \lim_{y \to 0} y^{\gamma} \sigma_{1xx}(h_1,y) = \frac{\mu_1\mu_2}{\sin\frac{\pi\gamma}{2}} \{-\frac{g_1(h_1)}{(2h_1)^{\gamma}} [\frac{1-2\gamma}{\mu_1+\kappa_1\mu_2} + \frac{1}{\mu_2+\kappa_2\mu_1}]$$
$$- \frac{g_2(h_2)}{(h_2-c)^{\frac{1}{2}}} [\frac{1-2\gamma}{\mu_2+\kappa_2\mu_1} + \frac{1}{\mu_1+\kappa_1\mu_2}]\} \;,$$

$$k_{xy} = \lim_{y \to 0} y^{\gamma} \sigma_{1xy}(h_1,y) = \frac{\mu_1\mu_2}{\cos\frac{\pi\gamma}{2}} \{-\frac{g_1(h_1)}{(2h_1)^{\gamma}} [\frac{1-2\gamma}{\mu_1+\kappa_1\mu_2} - \frac{1}{\mu_2+\kappa_2\mu_1}]$$
$$+ \frac{g_2(h_2)}{(h_2-c)^{\frac{1}{2}}} [\frac{1-2\gamma}{\mu_2+\kappa_2\mu_1} - \frac{1}{\mu_1+\kappa_1\mu_2}]\} \qquad (15)$$

In (15) the stress intensity factors k_{xx} and k_{xy} are defined explicitly because of their importance in the study of a possible delamination fracture at the interface. However, it should be noted that, because of (13), in k_{xx} and k_{xy} there is only one independent load factor, say $g_1(h_1)$ or $g_2(h_2)$. The solution of the system of singular integral equations with the generalized Cauchy kernels subject to the conditions (13) and (14) are obtained by using the technique outlined in [4,6].

Stress-free End Problem

The stress-free end problem is described in Figure 1c. Analytically the problem is simply the limiting case of the crack problem shown in Figure 1b in which $c \to 0$. In this case too first the problem for the whole plane $(-\infty<y<\infty)$ without the stress-free boundary $y = 0$ is solved and the homogeneous stresses given by (1) are determined. Then a perturbation problem with the end tractions

$$\sigma_{1yy}(x,0) = -p_1(x) \;, \quad \sigma_{2yy}(x,0) = -p_2(x) \;,$$

$$\sigma_{1xy}(x,0) = 0 \;, \quad \sigma_{2xy}(x,0) = 0 \qquad (16)$$

is considered. The end tractions are statically self-equilibrating and

satisfy (see (3))

$$\int_{-h_1}^{h_1} p_1(x_1)dx_1 + \int_{h_2}^{h_2} p_2(x_2)dx_2 = 0 \quad . \tag{17}$$

As in most cases, if p_1 and p_2 are constant, then

$$p_1h_1 + p_2h_2 = 0 \quad . \tag{18}$$

In this problem, basically the integral equations (5) are still valid with $a = h_1$, $b = h_2$, and $c = 0$. They may be solved by appropriately separating the generalized Cauchy kernels, defining

$$G_1(x_1) = g_1(x_1)(h_1^2-x_1^2)^{-\gamma} , \quad G_2(x_2) = g_2(x_2)(h_2^2-x_2^2)^{-\gamma} , \tag{19}$$

and using a technique described in [6]. Here, γ is obtained from the characteristic equation given by (12) and the end points of g_1 and g_2 are related by

$$g_2(h_2) = -(\frac{h_2}{h_1})^{\gamma} \frac{2\cos\pi\gamma + 4\alpha_2(1-\gamma)^2-\alpha_1-\alpha_2}{2\gamma(\alpha_1-\alpha_2) + 2-\alpha_1+3\alpha_2} g_1(h_1) \quad . \tag{20}$$

Again, to study a possible initiation of delamination fracture, it is useful to define and calculate the following stress intensity factors:

$$k_{xx} = \lim_{y\to 0} y^{\gamma}\sigma_{1xx}(h_1,y) = \frac{\mu_1\mu_2}{\sin\frac{\pi\gamma}{2}}\{-\frac{g_1(h_1)}{(2h_1)^{\gamma}}[\frac{1-2\gamma}{\mu_1+\kappa_1\mu_2} + \frac{1}{\mu_2+\kappa_2\mu_1}] - \frac{g_2(h_2)}{(2h_2)^{\gamma}}[\frac{1-2\gamma}{\mu_2+\kappa_2\mu_1} + \frac{1}{\mu_1+\kappa_1\mu_2}]\} ,$$

$$k_{xy} = \lim_{y\to 0} y^{\gamma}\sigma_{1xy}(h_1,y) = \frac{\mu_1\mu_2}{\cos\frac{\pi\gamma}{2}}\{-\frac{g_1(h_1)}{(2h_1)^{\gamma}}[\frac{1-2\gamma}{\mu_1+\kappa_1\mu_2} - \frac{1}{\mu_2+\kappa_2\mu_1}] - \frac{g_2(h_2)}{(2h_2)^{\gamma}}[\frac{1-2\gamma}{\mu_2+\kappa_2\mu_1} - \frac{1}{\mu_1+\kappa_1\mu_2}]\} \quad . \tag{21}$$

Numerical Results

The material constants used in the numerical examples and the corresponding powers of the stress singularity, β for a crack terminating at the interface as defined by (10) and γ for a crack crossing the interface (or for the stress-free end problem) as defined by (12) and (15) are given in Table 1. Here the material combinations A and B are the same and are assumed to approximate boron-epoxy sheets with buffer strips of the same material but different

stiffness.* The material pairs C and D correspond to Aluminum and Epoxy. Figures 2 and 3 give the stress intensity factor k_a for the material combinations A and B, respectively, where it is assumed that only material 1 contains a crack. Here, k_a is defined by the first equation of (9) for $a<h_1$ and by the second equation of (10) for $a=h_1$. Further results regarding this problem may be found in [2].

Table 1. The material constants and powers of stress singularity

	μ_1/μ_2	ν_1	ν_2	Plane Stress		Plane Strain	
				β	γ	β	γ
A	6.65	0.33	0.45	0.70148	0.16650	0.68856	0.22509
B	0.15	0.45	0.33	0.36210	0.16650	0.41539	0.22509
C	23.31	0.30	0.35	0.82503	0.21948	0.82562	0.27369
C	0.043	0.35	0.30	0.28873	0.21948	0.33795	0.27369

The stress intensity factors for the crack crossing the interface in the material combination A are given in Figures 4,5, and 6. The stress intensity factors k_c, k_{xx} and k_{xy} shown in the figures in normalized form are defined by (15). In the examples considered in Figures 2-6 it is assumed that the material is a thin plate (i.e., plane stress case) and is unconstrained in x direction. Hence, the constant tractions p_1 and p_2 used in the analysis are related by

$$\frac{p_1}{p_2} = \frac{E_1}{E_2} \quad . \tag{22}$$

Figures 7 and 8 show the results for the stress-free end problem described in Figure 1c for material combinations A and C, respectively. The figures give the thin plate results as well as the results for plane strain, i.e., for the layered medium which is "thick" in z direction. The stress intensity factors shown in these figures are defined by (21). In this case, stress-free boundary requires that the constant tractions p_1 and p_2 used in the analysis of the perturbation problem satisfy the equilibrium condition (18). The results given

*The problem for the orthotropic strips which is more representative of the composite sheets is currently being studied.

in Figures 7 and 8 are normalized with respect to $p_2\ell^{\gamma}$, where $\ell = (h_1+h_2)$. The magnitudes of p_1 and p_2 are determined from the full plane $(-\infty<y<\infty)$ under the specified external loads. For example, if the medium is subjected to a constant strain $\varepsilon^{\infty}_{1zz} = \varepsilon^{\infty}_{2zz} = \varepsilon_o$ in z direction, then

$$p_1 = \frac{\varepsilon_o E_1 E_2 h_2 (\nu_1 - \nu_2)}{h_2 E_2 (1-\nu_1^2) + h_1 E_1 (1-\nu_2^2)} , \quad p_2 = -\frac{h_1}{h_2} p_1 . \tag{23}$$

If the medium $(-\infty<x<\infty,\ 0<y<\infty)$ is subjected to uniform tension $\sigma_{1xx} = \sigma_{2xx} = \sigma_o$ in x direction, then

$$p_1 = \frac{\sigma_o h_2 (\nu_1 E_2 - \nu_2 E_1)}{h_2 E_2 (1-\nu_1) + h_1 E_1 (1-\nu_2)} , \quad p_2 = -\frac{h_1}{h_2} p_1 . \tag{24}$$

Or, if the temperature of the infinite medium $(-\infty<(x,z)<\infty,\ 0<y<\infty)$ is changed by an amount ΔT, then

$$p_1 = \frac{E_1 E_2 h_2 (\alpha_2 - \alpha_1) \Delta T}{h_2 E_2 (1-\nu_1) + h_1 E_1 (1-\nu_2)} , \quad p_2 = -\frac{h_1}{h_2} p_1 , \tag{25}$$

where α_1 and α_2 are the coefficients of thermal expansion of the layers 1 and 2 respectively. Input functions for other loading conditions may be obtained in a similar way. Needless to say, the results given in Figures 7 and 8 and those which may be obtained on the basis of the equilibrium conditions (17) or (18) are applicable to all these loading conditions.

Discussion and Conclusions

In Figure 2 for $a<h_1$ and $h_2 \to 0$ it may be seen that the stress intensity factor k_a becomes the value obtained from a homogeneous plane containing periodic collinear cracks, and for $a=h_1$ and $h_2 \to 0$, as expected, $k_a \to \infty$. Also, for $a \leq h_1$, as $h_2 \to \infty$, k_a approaches the value obtained for the strip bonded to two half planes, which is a special case of the problem under consideration [7,8]. These asymptotic values are shown in the Figures by dash-dot lines. Same results may be observed in Figure 3 where the crack is in the less stiff material. Here, for $a<h_1$ and $h_2 \to 0$, k_a must and does approach the same periodic crack values shown in Figure 2. However, the stress intensity factors in this case are much lower than those given in Figure 2 where the crack is in the stiff material.

In Figure 4, for $c \to 0$, as expected, $k_c \to \infty$. It is seen that $k_c \to \infty$ also when $c \to h_2$. The reason for this is that for $c=h_2$ the power of stress singularity β is greater than 0.5 (material combination A), whereas k_c shown in

Figure 4 is calculated on the basis of one half power singularity. It may also be observed that k_c becomes greater as h_1/h_2 increases. Since the power of the stress singularity β for a crack terminating at the interface is always greater than γ for a crack crossing the interface, k_{xx} and k_{xy} also go to infinity as the crack tip approaches the interface (Figures 5 and 6).

When the singular point is at the interface as in problems for a crack touching or crossing the interface, since the power of singularity is not 0.5, for a crack propagation initiating at this point the stress state around the crack tip does not remain self-similar. Therefore, in this case most of the conventional fracture and fatigue models do not seem to be applicable. In the absence of a physically more acceptable criterion, at present one may assume that for this type of fracture problems the "maximum stress criterion" is adequate. This criterion may be stated as "the fracture propagation will take place radially in the direction $\theta = \theta_c$ for which the cleavage stress $\sigma_{\theta\theta}$ is maximum and when

$$\sigma_{\theta\theta}(\delta_p, \theta) \geq \sigma_c = \text{constant}, \tag{26}$$

where δ_p is the size of the fracture process zone around the crack tip". The process zone size depends on the microstructure and continuum properties of the material as well as on the environmental conditions. The critical stress σ_c is considered to be a "material constant" representing the cohesive strength of the constituent materials and, for the appropriate δ_p, is determined from controlled experiments for an idealized geometry and loading. If the "weak link" around the singular point is the interface, then (26) may be modified as follows

$$(\sigma_c)_{12} \leq \begin{cases} (\sigma_{\theta\theta}^2 + \sigma_{r\theta}^2)^{\frac{1}{2}} \quad , & \sigma_{\theta\theta} > 0 \\ \sigma_{1\theta} + f\sigma_{r\theta} \quad , & \sigma_{\theta\theta} < 0 \end{cases} \tag{27}$$

where $(\sigma_c)_{12}$ represents the adhesive strength of the bond, $f\sigma_{\theta\theta}$ is the friction resistance, and the stresses $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ are calculated at a distance $r = (\delta_p)_{12}$ from the singular point and represents the process zone size for the particular joint.

Finally, from Table 1 and Figures 7 and 8 it may be observed that in the stress-free end problem the stress intensity factors, k_{xx}, k_{xy} and the power of the stress singularity γ for the plane stress conditions are greater than those for the plane strain case. This means that for the stress-free end

problem shown in Figure 1c the stress state around the bonded corners under plane stress conditions is expected to be much more severe than that under plane strain conditions.

Acknowledgment

This work was supported by the Materials Division, NASA-Langley under the Grant NGR 39-007-011 and by the Engineering Division of NSF under the Grant ENG 73-045053 A01.

References

1. Eisenmann, J. R. and Kaminski, B. E., "Fracture Control for Composite Structures", *J. Engineering Fracture Mechanics*, Vol. 4, p. 907, 1972.

2. Erdogan, F. and Bakioglu, M., "Fracture of Plates which Consist of Periodic Dissimilar Strips", *Int. J. Fracture*, Vol.12, p.   , 1976.

3. Muskhelishvili, N. I., *Singular Integral Equations*, P. Noordhoff, Groningen, Holland, 1953.

4. Erdogan, F. and Gupta, G. D., "On the Numerical Solution of Singular Integral Equations", *Quart. Appl. Math.*, Vol. 30, p. 525, 1972.

5. Erdogan, F. and Bincikoglu, V., "Two Bonded Half Planes with a Crack Going Through the Interface", *Int. J. Engng. Sci.*, Vol. 11, p. 745, 1973.

6. Erdogan, F., "Complex Function Technique", *Continuum Physics*, Vol. II, A. C. Eringen (ed.), Academic Press, p. 523, 1975.

7. Bogy, D. B., "The Plane Elastostatic Solution for a Symmetrically Loaded Crack in a Strip Composite", *Int. J. Engng. Sci.*, Vol. II, p. 985, 1973.

8. Gupta, G. D., "A Layered Composite with a Broken Laminate", *Int. J. Solids, Structures*, Vol. 10, p. 1141, 1973.

APPENDIX

Referring to (5) and (6), let the asymptotic expression of K_{ij} for $s\to\infty$ be K_{ijs}, i.e.,

$$K_{ij}(x_i,t,s) = K_{ijs}(x_i,t,s) + K_{ijf}(x_i,t,s) \ , \ (i,j=1,2) \ . \tag{A1}$$

Then defining

$$k_{ij}(x_i,t) = k_{ijs}(x_i,t) + k_{ijf}(x_i,t) \ , \ (i,j=1,2) \ , \tag{A2}$$

the singular parts, k_{ijs} and bounded parts, k_{ijf} of the kernels k_{ij} can be obtained by substituting from (A1) and (A2) into (6). Thus, using the expressions for K_{ij} given in [2], the singular kernels k_{ijs} are found to be:

$$\begin{aligned}
k_{11s}(x_1,t) &= \left[\frac{3\alpha_2-\alpha_1}{2} + 6\alpha_2(h_1+x_1)\frac{d}{dx_1} + 2\alpha_2(h_1+x_1)^2\frac{d^2}{dx_1^2}\right][t-(2h_1+x_1)]^{-1} \\
&+ \left[\frac{3\alpha_2-\alpha_1}{2} - 6\alpha_2(h_1-x_1)\frac{d}{dx_1} + 2\alpha_2(h_1-x_1)^2\frac{d^2}{dx_1^2}\right][t-(2h_1-x_1)]^{-1} \ , \\
k_{12s}(x_1,t) &= \left[(\alpha_1-\alpha_2)(h_1-x_1)\frac{d}{dx_1} + \frac{2-\alpha_1+3\alpha_2}{2}\right][t-(h_1+h_2-x_1)]^{-1} \\
&+ \left[(\alpha_2-\alpha_1)(h_1+x_1)\frac{d}{dx_1} + \frac{2-\alpha_1+3\alpha_2}{2}\right][t-(h_1+h_2+x_1)]^{-1} \ , \\
k_{21s}(x_2,t) &= \left[(\alpha_3-\alpha_4)(h_2-x_2)\frac{d}{dx_2} + \frac{2-\alpha_3+3\alpha_4}{2}\right][t-(h_1+h_2-x_2)]^{-1} \\
k_{22s}(x_2,t) &= \left[\frac{3\alpha_4-\alpha_3}{2} - 6\alpha_4(h_2-x_2)\frac{d}{dx_2} + 2\alpha_4(h_2-x_2)^2\frac{d^2}{dx_2}\right][t-(2h_2-x_2)]^{-1} \ .
\end{aligned} \tag{A3}$$

$$\begin{aligned}
\alpha_1 &= (\kappa_1\mu_2-\kappa_2\mu_1)/(\mu_2+\kappa_2\mu_1) \ , \quad \alpha_2 = (\mu_2-\mu_1)/(\mu_1+\kappa_1\mu_2) \ , \\
\alpha_3 &= (\kappa_2\mu_1-\kappa_1\mu_2)/(\mu_1+\kappa_1\mu_2) \ , \quad \alpha_4 = (\mu_1-\mu_2)/(\mu_2+\kappa_2\mu_1) \ .
\end{aligned} \tag{A4}$$

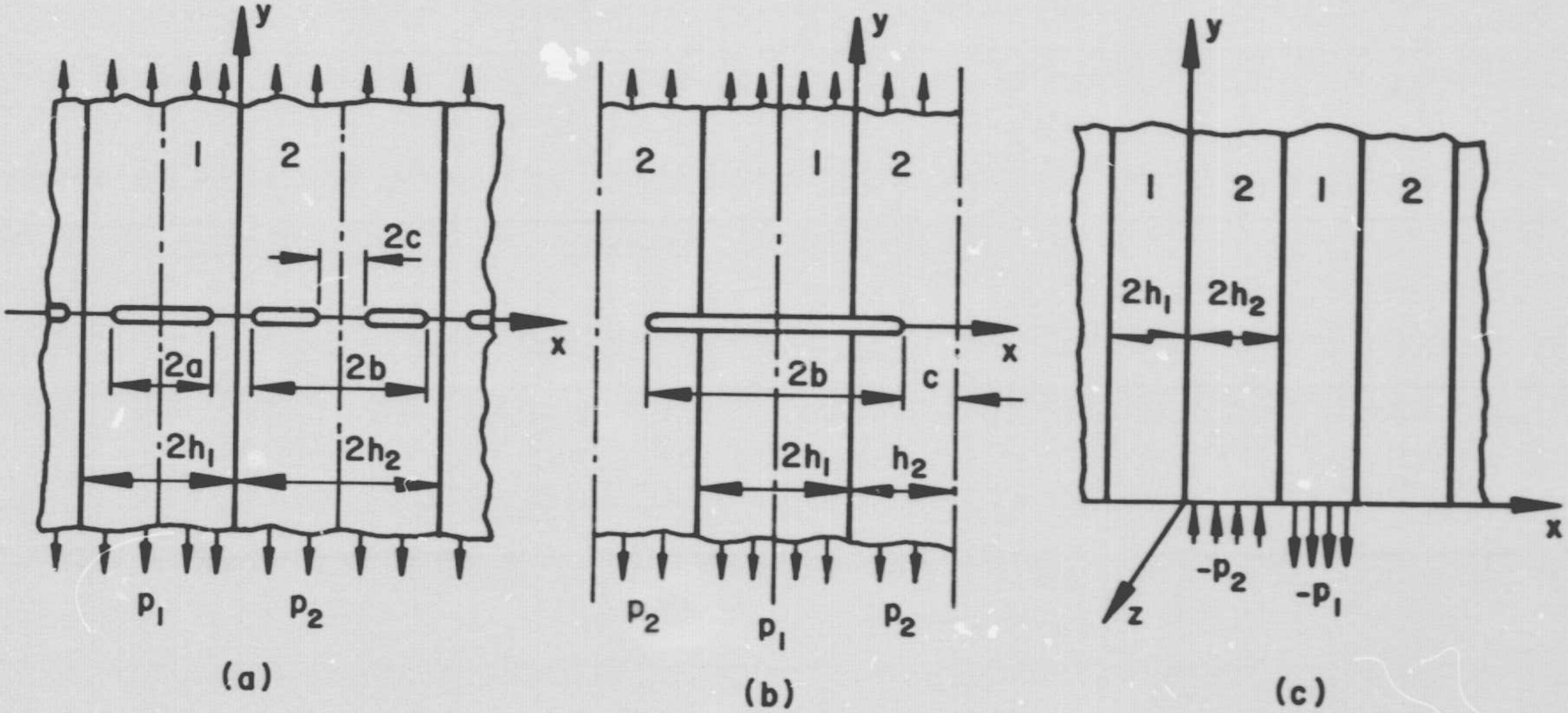

Figure 1. The geometry of bonded strips and the stress-free end.

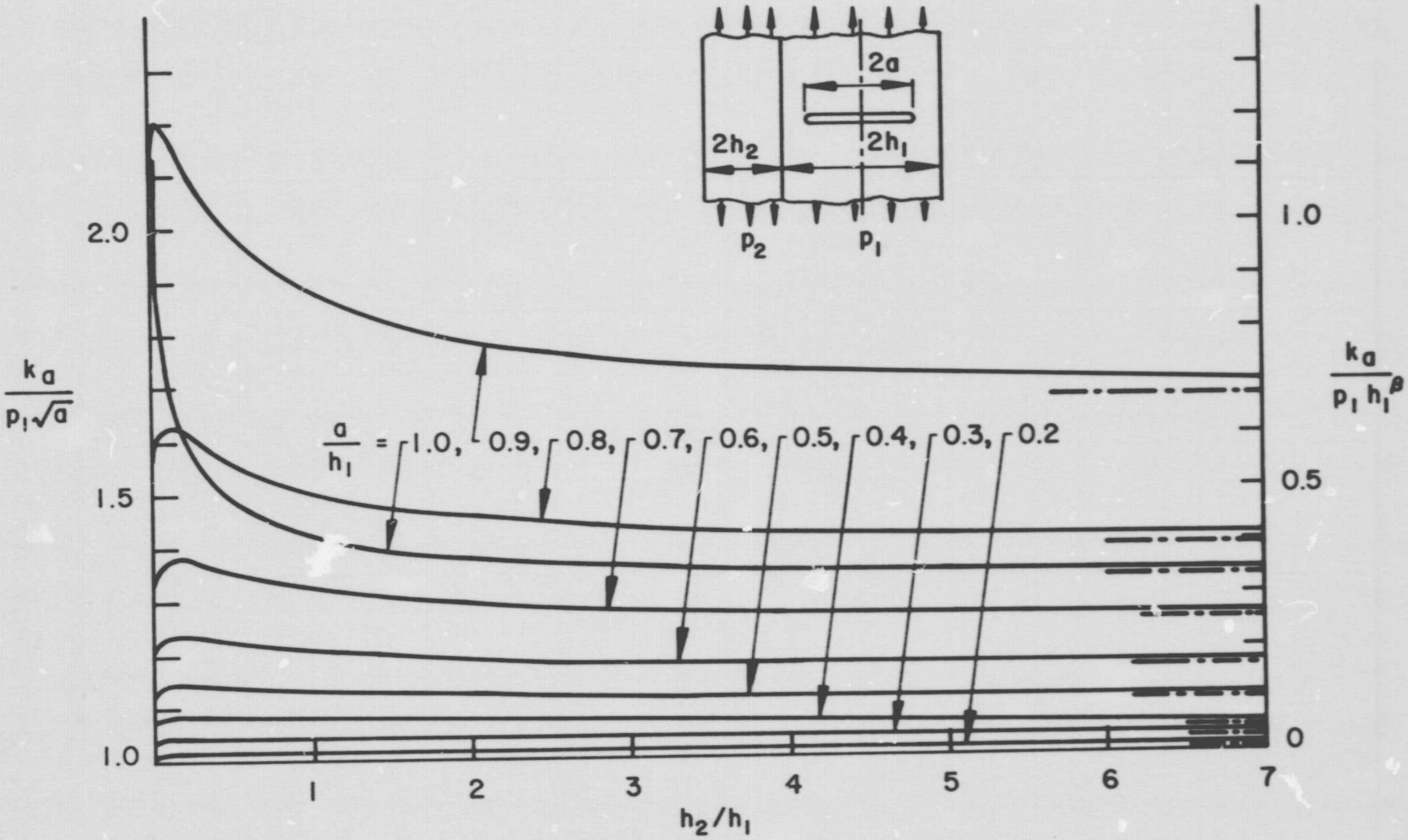

Figure 2. Stress intensity factor in bonded strips for the crack in the stiffer material ($\mu_1 = 6.65\mu_2$, $\nu_1 = 0.33$, $\nu_2 = 0.45$; the scale on the right corresponds to the broken strip case, i.e., to $a = h_1$; dash-dot lines give the asymptotic values for $h_2 \to \infty$).

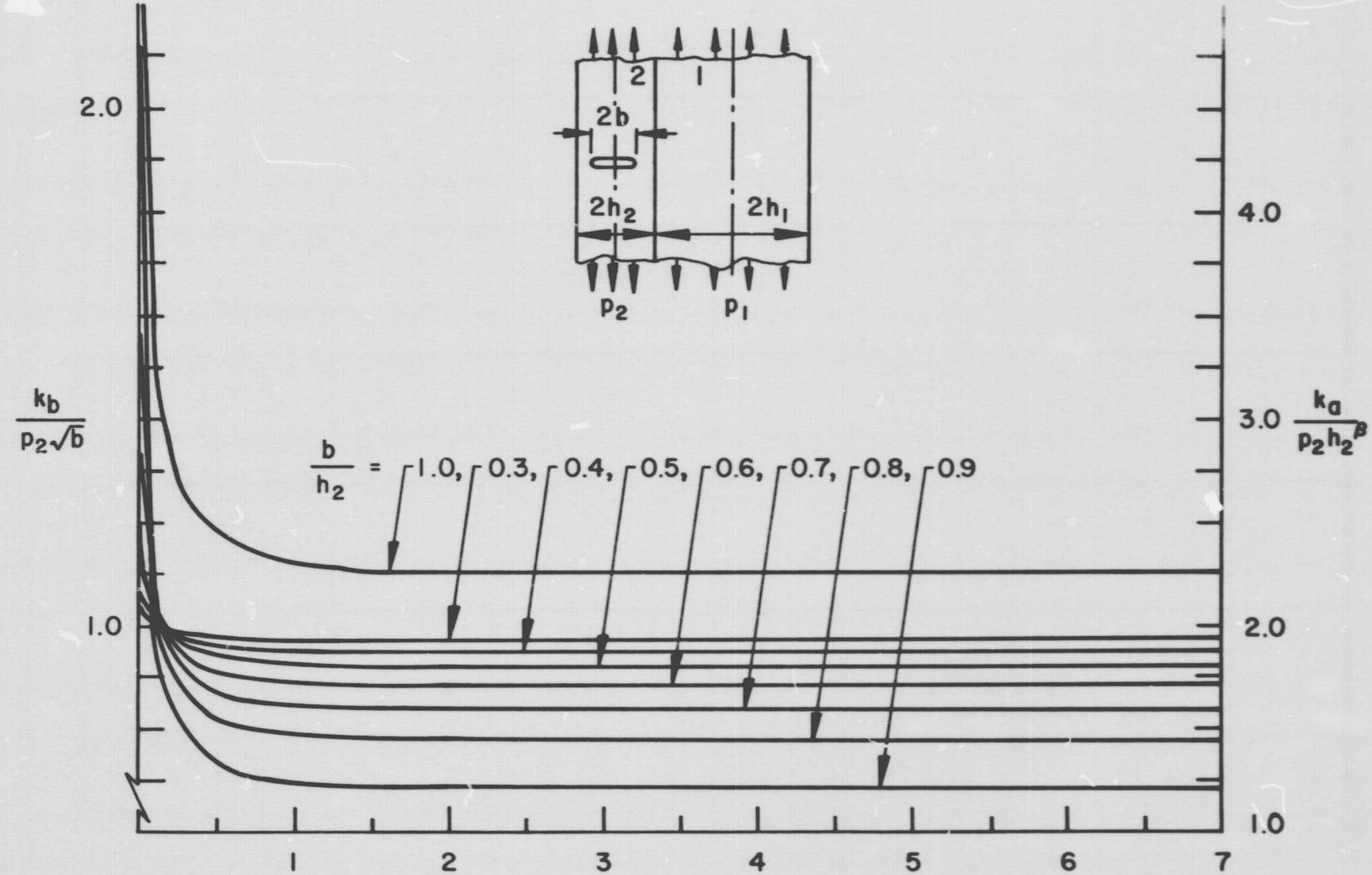

Figure 3. Stress intensity factor in bonded strips for the crack in less stiff material ($\mu_1 = 6.65\mu_2$, $\nu_1 = 0.33$, $\nu_2 = 0.45$; the scale on the right corresponds to $b = h_2$).

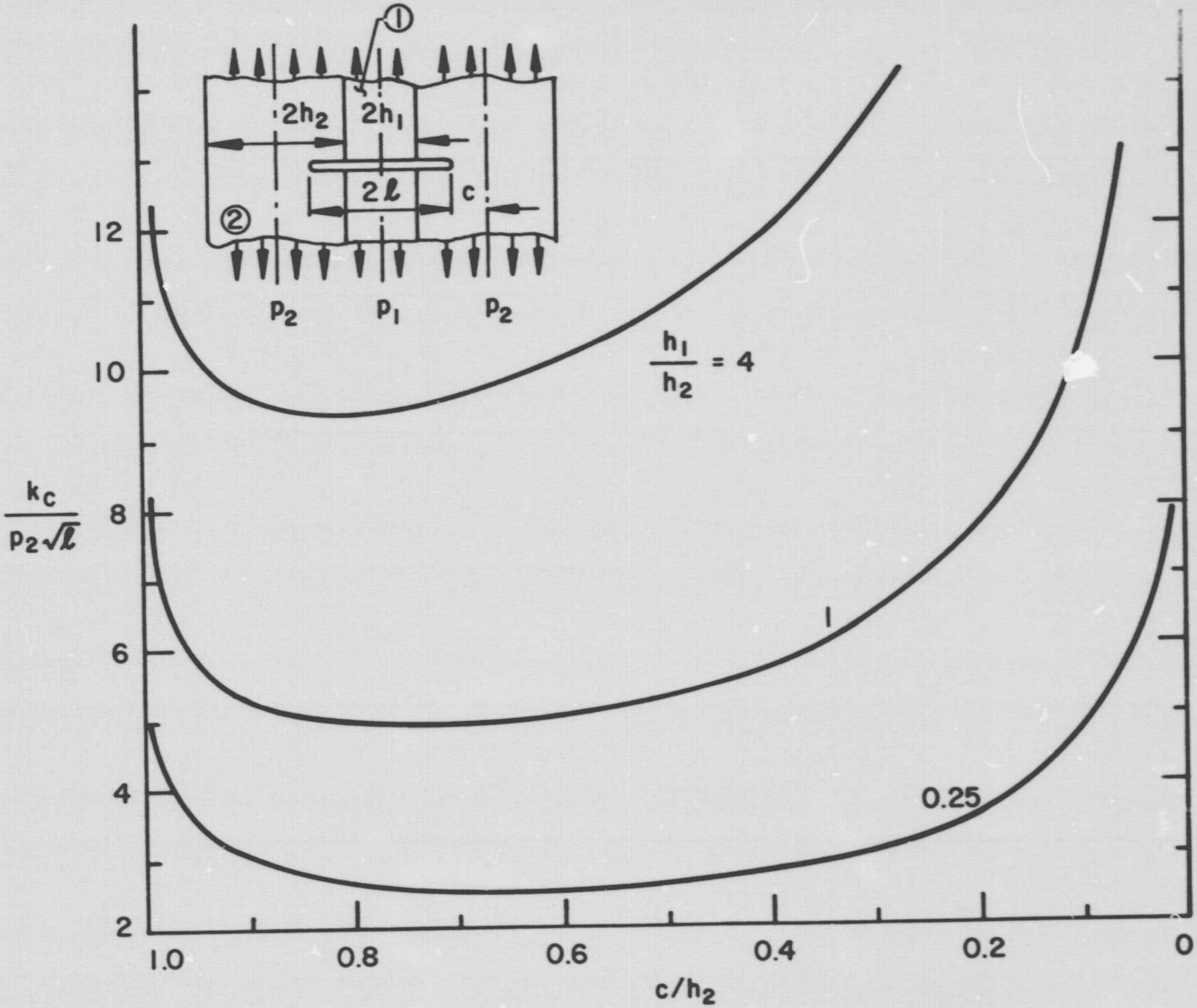

Figure 4. Stress intensity factor at the crack tip in bonded strips for the crack crossing the interface where material 1 is fully cracked ($\mu_1 = 6.65\mu_2$, $\nu_1 = 0.33$, $\nu_2 = 0.45$, $\ell = h_1+h_2-c$).

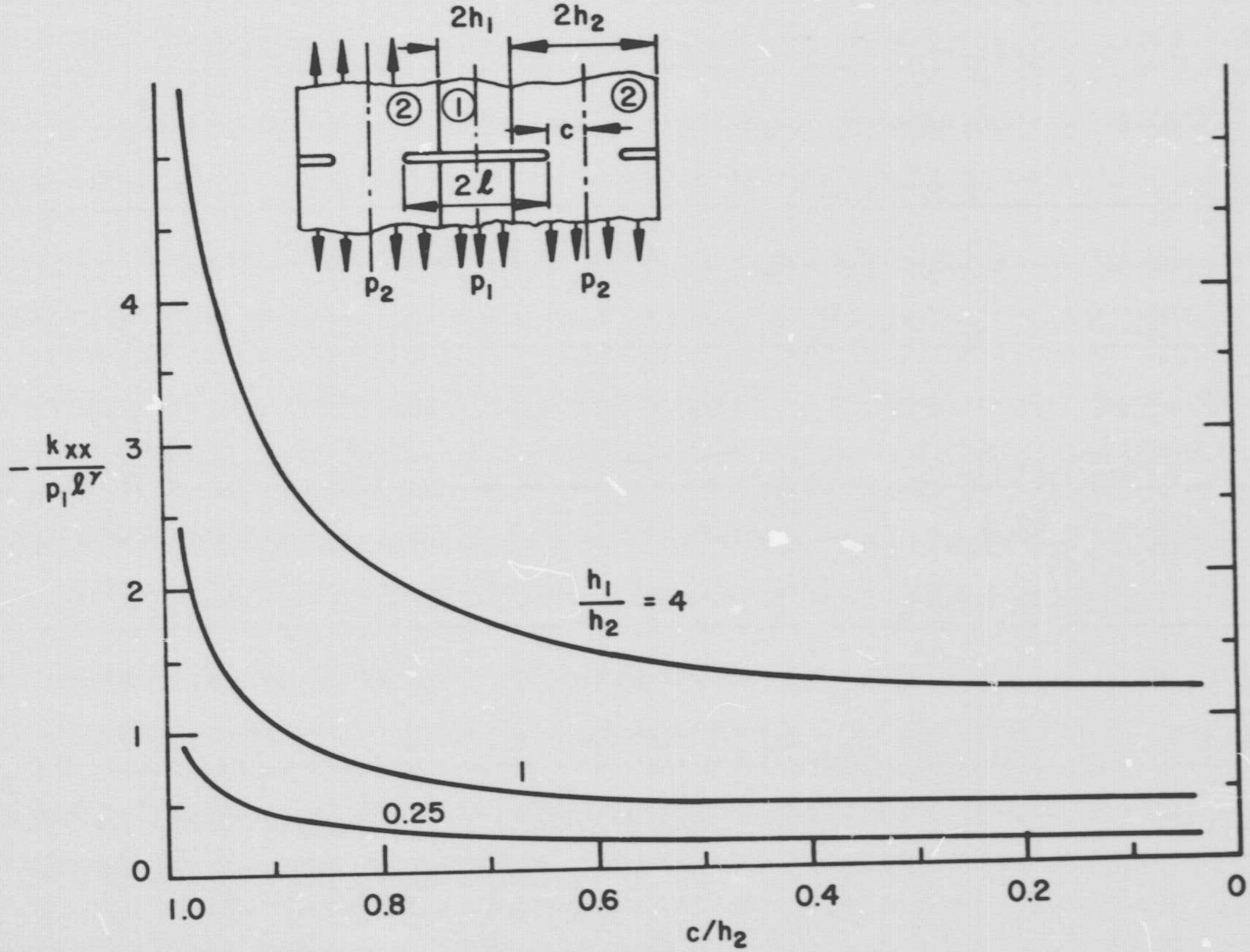

Figure 5. Normal component k_{xx} of the stress intensity factor at the intersection of the crack and the interface for the crack crossing the interface where material 1 is fully cracked ($\mu_1 = 6.65\mu_2$, $\nu_1 = 0.33$, $\nu_2 - 0.45$, $\ell = h_1+h_2-c$).

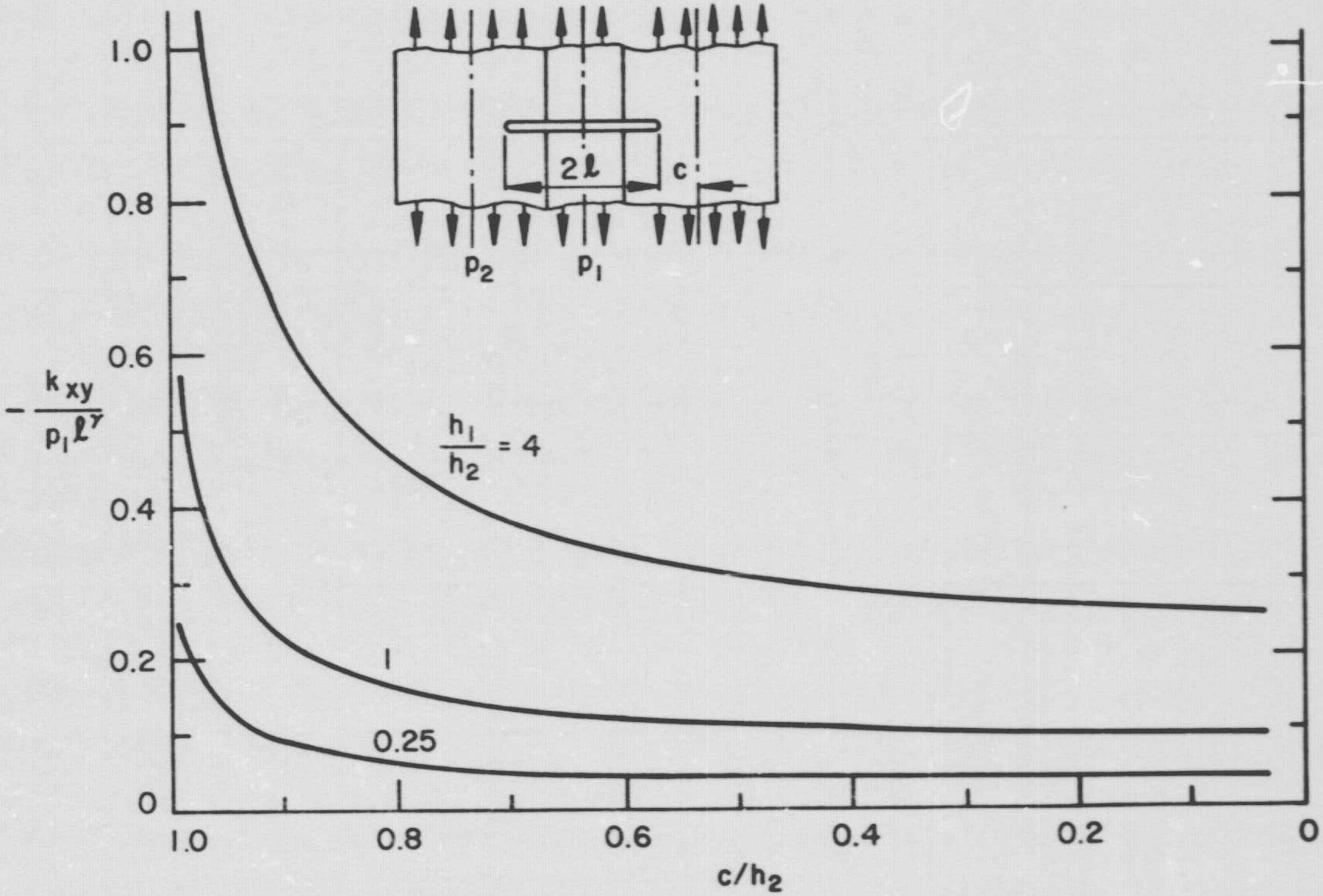

Figure 6. Shear component k_{xy} of the stress intensity factor at the intersection of the crack and the interface for the crack crossing the interface where material 1 is fully cracked ($\mu_1 = 6.65\mu_2$, $\nu_1 = 0.33$, $\nu_2 = 0.45$, $\ell = h_1 + h_2 - c$).

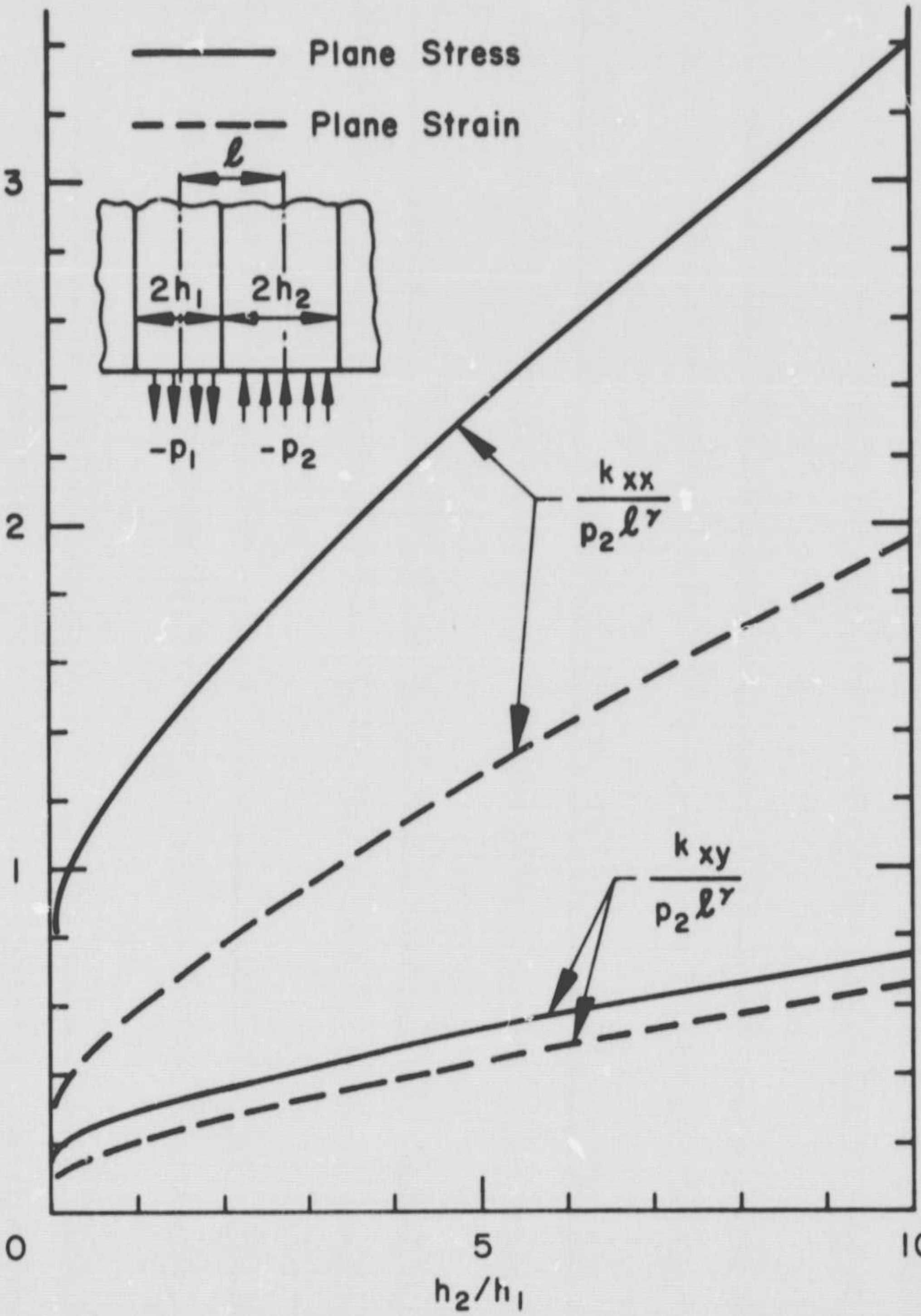

Figure 7. Normal and shear components of the stress intensity factor, k_{xx} and k_{xy} at the intersection of the interface and the stress-free boundary in a layered half plane; $\mu_1 = 6.65\mu_2$, $\nu_1 = 0.33$, $\nu_2 = 0.45$, $\ell = h_1 + h_2$.

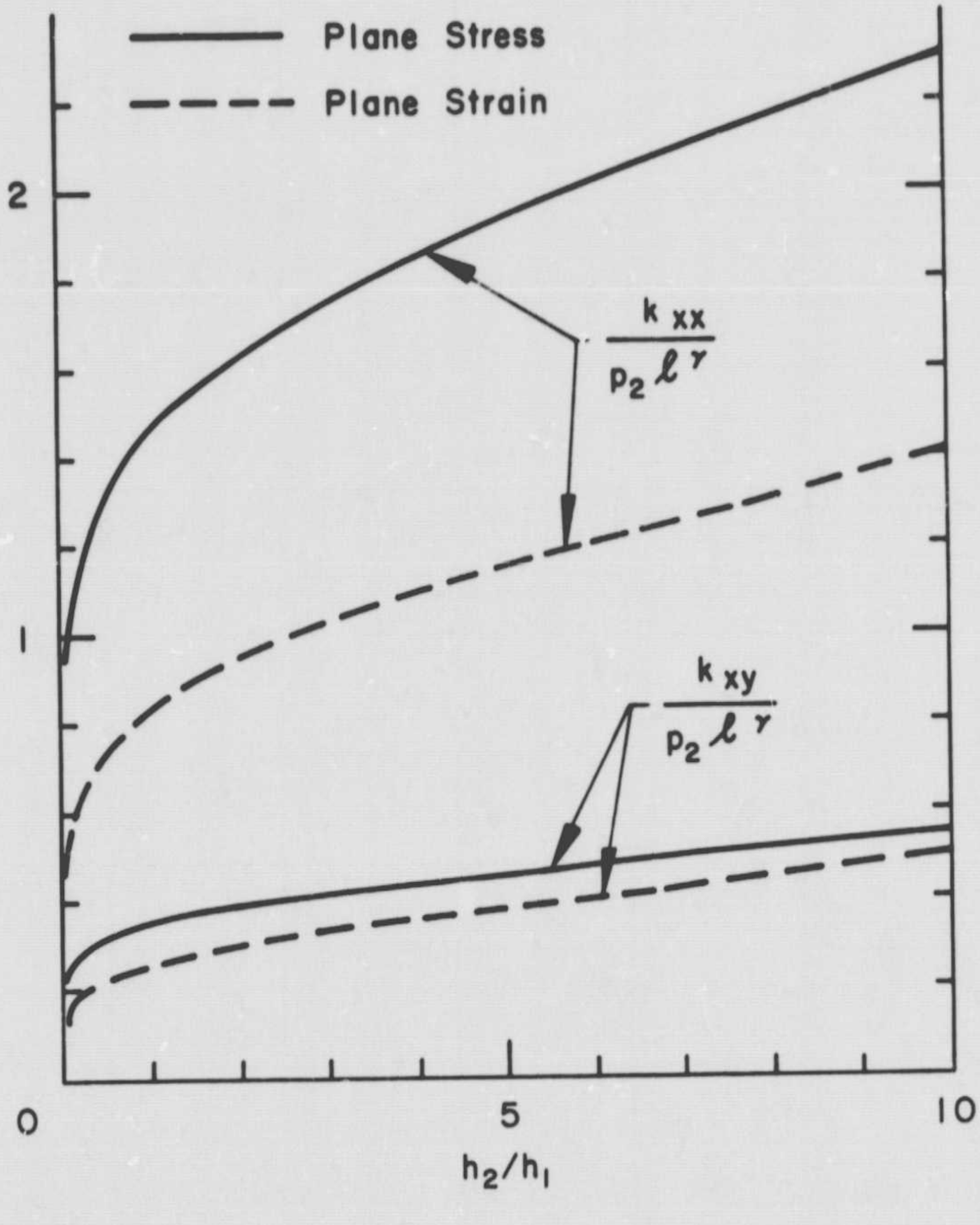

Figure 8. Normal and shear components of the stress intensity factor, k_{xx} and k_{xy} at the intersection of the interface and the stress-free boundary in a layered half plane; $\mu_1 = 23.31\mu_2$, $\nu_1 = 0.3$, $\nu_2 = 0.35$, $\ell = h_1+h_2$.